THE PRINCIPLE OF RELATIVITY.

INTRODUCTORY.

PHYSICAL science seems to have entered into a new phase, the slogan of the new school being THE PRINCIPLE OF RELATIVITY. In some quarters the current modes of thought are declared antiquated, and the promise is made that the old truths will acquire a new meaning. Physicists speak of the relativity of time and space, and we will add that they ought as well speak of the relativity of things, of the whole actual world in all its parts and interrelations.

Many who have watched the origin and rise of the new movement are startled at the paradoxical statements which some prominent physicists have made, and it is remarkable that the most materialistic sciences, mechanics and physics, seem to surround us with a mist of mysticism. The old self-contradictory statements of the Eleatic school revive in a modernized form, and common sense is baffled in its attempt to understand how the same thing may be longer and shorter at the same time, how a clock will strike the hour later or sooner according to the point of view from which it is watched; and the answer of this most recent conception of physics to the question, How is this all possible? is based on the principle of the relativity of time and space.

The man who started this movement and was the first to formulate it in concise language and to base it upon close

argument was Professor Einstein,[1] who was followed by Lorentz,[2] and so we hear often of the Einstein-Lorentz theory. The strangest thing about it is that the question is seriously debated whether or not this theory is true, and the answer is expected from experiments; while in our opinion we are here confronted with a method, and the problem is simply how we can best deal with certain difficulties due to the relativity of all things. These difficulties have originated through the need of a greater exactness in measurements, but the underlying truth—the relativity of all things—is not a question of fact, but a recognition of certain complications with which we must learn to deal.

On reading recent expositions of the principle of relativity the man of good education, or the one who has attended universities without being a specialist in either mathematics or physics, feels the *terra firma* give way under his feet, and when he finds that the principle of identity seems to fail in his comprehension of things, a dizziness comes over his intellect and he sinks into the bottomless abyss of the incomprehensibility of existence. A general earthquake seems to quiver through his mind. Everything totters around him and he stands in awe at the significance of the new thought. Nor is there any one who dares to contradict; for the most learned arguments are adduced, the mathematical and logical conclusions of which bristle with formidable formulas,—yea, experiments are made to prove the truth of the relativity of time and space.

For the sake of convenience we will speak of the representatives of this new conception as the "relativity physicists" in contradistinction to the old-fashioned physicists of the old school. It has been said that the former represent more the mathematical aspect of physics while the

[1] *Jahrbuch der Radioaktivität und Elektronik,* 1905-1908.

[2] H. A. Lorentz, *Theory of Electrons* (Teubner) 1910.

latter are the realistic physicists proper, too realistic to understand the significance of the new truth.

In order to facilitate a comprehension of the situation as well as our own conception, we will here at once and dogmatically state that the relativity physicists are perfectly right; what they claim is really and truly a matter of course, and if they only would present their proposition without dressing up their theory in paradoxical statements, nobody would in the least hesitate to accept the new view. But as soon as this is done people will at the same time find out that the new view is not novel. Its importance has been greatly exaggerated, for the principle has been tacitly understood in the correct way by all preceding physicists who, at the time however, ignored, or better did not enter into, the problem, because they had other more pressing work on hand. Nor is it unlikely that they regarded this problem of relativity as a philosophical question which strictly speaking had no place before the forum of physics.

ON THE ABSOLUTE.

Perhaps the easiest way of elucidating the true meaning of the relativity of time and space will be by setting forth our own position as we held it long before the principle of relativity gained prominence or had even been mentioned or alluded to.

The writer's book *Fundamental Problems* contains the following statement under "Definitions and Explanations" (first edition, page 254; seecond edition, page 252):

"Absolute existence (in fact everything absolute) is impossible. Reality is properly called *Wirklichkeit* in German, derived from *wirken,* to take effect. Reality is not immovable and unchangeable absoluteness, but the effectiveness of things in their relations. Reality therefore implies not only existence, but the manifestation of existence

also. Existence and its manifestation are not two different things; both are one."

Since the days of Heraclitus it has been a trite truism that all existence is in a flux. There is no rest anywhere, and actuality consists in the effects which these changes exercise upon one another by action and reaction. Upon this lack of stability, resulting from a universal and intrinsic relativity, Mr. Spencer bases one of the strongest, though quite untenable, arguments of his agnosticism. He seems to expect that time, space, motion, and matter are or should be things-in-themselves, and forgets that they represent relations, i. e., certain features of reality. We will here quote his exposition of the unknowableness of motion in space. In his *First Principles* Spencer says:

"Here, for instance, is a ship which, for simplicity's sake, we will suppose to be anchored at the equator with her head to the west. When the captain walks from stem to stern, in what direction does he move? East, is the obvious answer,—an answer which for the moment may pass without criticism. But now the anchor is heaved, and the vessel sails to the west with a velocity equal to that at which the captain walks. In what direction does he now move when he goes from stem to stern? You cannot say east, for the vessel is carrying him as fast towards the west as he walks to the east; and you cannot say west for the converse reason. In respect to surrounding space he is stationary; though to all on board the ship he seems to be moving. But now are we quite sure of this conclusion? Is he really stationary? When we take into account the earth's motion round its axis, we find that instead of being stationary he is traveling at the rate of 1000 miles per hour to the east; so that neither the perception of one who looks at him, nor the inference of one who allows for the ship's motion, is anything like the truth. Nor indeed, on further consideration, shall we find this revised conclusion to be much better. For we have forgotten to allow for the earth's motion in its orbit. This being some 68,000 miles per hour it follows that, assuming the time to be midday, he is moving, not at the rate of 1000 miles per hour to the east, but at the rate of 67,000 miles per hour to the west. Nay, not even now have we discovered the true rate and the true direction of his movement. With the

earth's progress in its orbit, we have to join that of the whole solar system towards the constellation of Hercules; and when we do this, we perceive that he is moving neither east nor west, but in a line inclined to the plane of the ecliptic, and at a velocity greater or less (according to the time of the year) than that above named. To which let us add, that were the dynamic arrangements of our sidereal system fully known to us, we should probably discover the direction and rate of his actual movement to differ considerably even from these. How illusive are our ideas of motion, is thus made sufficiently manifest. That which seems moving proves to be stationary; that which seems stationary proves to be moving; while that which we conclude to be going rapidly in one direction, turns out to be going much more rapidly in the opposite direction. And so we are taught that what we are conscious of is not the real motion of any object, either in its rate or direction; but merely its motion as measured from an assigned position—either the position we ourselves occupy or some other."

The same argument of the captain walking the deck of a ship was made before Spencer, though mostly it was a ball rolling on deck; Bradley refers to it as well known in his time, 1727, and the same story has been repeated after Spencer. In fact it is one of the arguments of the relativity of space among modern relativity physicists.

The principle upon which the representatives of the new view take their stand is a consideration of actual life. Things are in a flux, and this is an undeniable fact. We must bear in mind that the way of making knowledge possible at all in the flux of being is to ignore what has nothing to do with the problem under investigation. Our method is based upon a fiction or, if you please, upon an artificial trick, viz., to ignore complications and to consider a certain thing as fixed; but there are cases in which we must remember that we ourselves change and that the very position we assume is moving.

This trick of assuming that our position is stable is easy enough because man does not at once notice that there is any change; but all things are in a flux and he himself

changes unconsciously. A primitive unsophisticated man does not know that the earth on which he stands is whirling around itself at the rate of 1037 miles an hour, on the equator, further that it is also revolving with incredible speed around the sun, and that with the sun it is proceeding in a spiral motion towards one of the constellations, probably the constellation Heracles, around an unknown center situated somewhere in the Milky Way. God only knows what else takes place and what kind of whirling dances the Milky Way performs. The savage has not the slightest idea of all this, and so it is easy for him to ignore the motion of which he unconsciously partakes.

If man really were aware of all the events which influence him, his head would swim, and he would be incapable of thinking any sober thought. Fortunately he is concerned solely with his own narrow interests. The more man in the further growth of his mind becomes familiar with these unnoticeable events, the more he discovers that for any particular purpose he must ignore what does not belong to the solution of the special problem under consideration.

This way of ignoring what does not concern us at the time is an artificial process, a process of abstraction and elimination, of cutting off all disturbing incidents, and in doing so the philosophically minded scientist will become aware of the fiction of arbitrarily laying down a point of reference which is treated as if it were stable while in fact, like everything else, it too is caught in the maelstrom of cosmic existence.

There is nothing wrong or harmful in this fiction; on the contrary it is an indispensable part of our method of comprehending things. The universe is too complicated to be understood or viewed at a glance, and knowledge, science, cognition as well as all mental processes become possible merely by concentration, i. e., by selecting a point of

view as being a certain fixed location from which we observe a change, an event, a transformation, in order to gain a comprehension of this or that piece of existence in contrast to others of the same or of a different kind. Such is the nature of cognition, and this artificial trick is an essential condition of observation.

Knowledge is relative. It is the relation between subject and object, the thinker and the thing, and this, far from being objectionable, is only the universal condition of all existence; for all existence is relative. All reality is the result of action and reaction; it is a forming and being formed under definite conditions; it is transformation. There is no existence in and by itself. Relativity is the principle of all real and actual being.

TRICKS OF COGNITION.

If the standpoint of an observer changes, the thing observed will naturally change too in its relation to him. Formerly physicists were in the habit of not seriously bearing in mind that the fixedness of their standpoint was an assumption; they did not follow this principle to its ultimate consequences. For their special problems it was not necessary to do so, and there is very little use in bearing it constantly in mind. The difference in time between the moment when the observer looks at an object and that in which the rays of light indispensable for observation strike his eye is too inconsiderable to be taken into account; it is a negligible quantity. But if the object under consideration is at such an enormous distance that it takes the rays of light thousands of years to reach the eye of the astronomer it does make a difference, and so James Bradley was astonished to register the fact that the fixed stars in the sky were not always in the same place but that they pendulated semi-annually above us with the motion of the earth around the sun. The direction in which we see them swings from

the aphelion to the perihelion, and a closer consideration of the facts shows that the rays of very distant stars which we catch in the aphelion are not caused at the moment when we see them but started thousands of years prior to the moment in which they strike the lens of the astronomer's telescope, and so the transference of rays of light from the star to the astronomer's eye at this enormous distance represents a relation which most forcibly drives the truth home to us that there is nothing absolute.

The same is true of all things. The object before us seems to stand there in a perfect and quiet completeness, and yet the changes that work unnoticed by our dull senses are constant, continuous and rapid. Heraclitus used to say that he could not come out of the same river into which he had stepped a moment before, because the water was always rushing by. Never is a drop of it the same, and this is true of all things, even of ourselves. The observer has to exclude from his methods of observation the fact that he himself, his senses and his mind, are in a constant flux.

In order to elucidate the significance of the nature of cognition as being a limitation and concentration upon one point and constructing artificial units, the writer has on former occasions used the analogy of the kinematoscope, the machine which produces moving pictures.

In order to make any picture possible we need a lens, and the lens focuses the rays of light so as to throw rays from the same spot upon one and the same place on the plane where the picture appears. The rays of light which proceed from an object scatter in all directions, and unless we use a lens to concentrate the rays, the formation of a picture of the object would remain impossible. Thus the method of producing a picture is by concentration.

The lens produces a picture by focusing rays of light, that is by throwing the same rays upon the same spot; but it would also be possible to produce a picture by cutting

off the redundant rays of light and singling out one or very few rays, each one coming from each of the several points of the object. Accordingly we can photograph objects through a pinhole; there is only this difference that the picture is weak and needs long exposure. This proves that the process of concentration is fundamentally a process of abstraction, of leaving out, of omitting the disturbing multiplicity of the innumerable facts of real life as represented in the totality of objective experience.

The kinematoscope involves not only the static form of things, their spatial expression, the juxtaposition of parts, but it also adds the changes that are taking place in time. The film of the kinematoscope consists of a series of pictures, one always a little different from another, and if these are presented in rapid succession the series is fused into one picture in which the succeeding differences appear as motion. This is accomplished by the introduction of a little winged wheel which in rapid succession covers and uncovers the several pictures. If we would take this little wheel with its wings out of the kinematoscope, and if otherwise the pictures on the film would succeed one another in a rapid continuous motion without this artificial separation by the wings of the wheel, we would see no picture at all but simply have a blur on the canvas. In order to have distinct pictures appear on the canvas, we must cut the flux of motion into little separate moments which we may allegorically characterize as atoms of time.

Reality is a continuous flux, but in order to follow it step by step we must do the same thing that the mathematician does with his differential calculus. In the calculus the curve is cut up into infinitesimal lines, which in continuous succession change their directions, and the smaller we conceive these lines to be, the less is the mistake made by this fiction, if they are treated like straight lines.

The method of the calculus, based upon the fiction of

substituting for a continuous curve a series of little straight lines constantly changing their direction, is not so very different from the method of cognition in general. Nor is there anything wrong in it, only we must remain conscious of the fiction. In a similar way we must know that existence itself is a continuous system of relations, or in other words, that relativity is the principle of all existence in the world of actual life as well as in the domain of thought. We must cut up the general flux according to the needs of our investigation and lay down artificial limits.

* * *

If we view the new physics under this aspect, it will lose its mystic glamor and at the same time appear intelligible. In fact we shall understand that the principle of relativity is a matter of course, and if we cut up reality into things, as if they were things-in-themselves, into units or atoms, we employ a trick of cognition which makes it possible to focus things and picture them distinctly in our mind.

There are large numbers of scientists possessed of an *odium philosophicum* because philosophy means to them some abstruse metaphysical system of thought which ignores the natural sciences and, spiderlike, spins a world-conception out of pure thought derived from the thinker's subjectivity. The result is that they are soon perplexed in their own science by philosophical problems; for true philosophy—the philosophy of science—is an indispensable factor of cognition, and its influence extends into the fabric of all scientific labors. Thus it happens that problems of a philosophical character arise unexpectedly, and then the information given by nature in reply to experiments is apt to be misunderstood.

If the reference point (R) from which an observer measures is in motion toward R_1, and the object observed (O) also possesses a motion of its own, we are confronted

with a complicated phenomenon. If R moves toward O, the object measured will be shorter than if it stands still, and it will be longer if R moves with O in the same direction. We have only to forget, after the fashion of the pragmatist, that there is an ideal of objective cognition, and assume that all there is about size or the objective measure of things consists in the result of our measuring and we have the clue to the paradoxes of the physics of relativity. If the point of reference is not stationary and if we neglect to account for its motion, the result of our measurement is necessarily vitiated thereby as much as the pragmatist's philosophy by his personal equation.

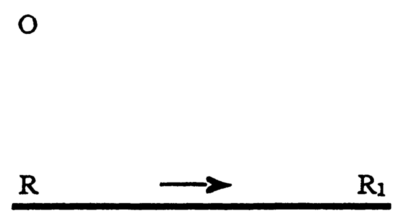

Fig. 1.

There are further complications of measurement. The time needed for the transmission of signals must also be taken into consideration. The rays of light travel at an enormous velocity but the distances in the starry heavens are also enormous and the distance between O and R is less than between O and R_1. The rays which were sent out from O at the moment of measurement have already passed the track of the observer at R, while this same observer has moved on to R_1, and there he catches the rays sent out from O in its position at O; in the meantime however the object O has in its turn also changed its place. From R_1 it appears at O, where it stood while the observer was stationed at R, but in fact it stands no longer at O but has in the meantime proceeded on its own path whithersoever that may have led O, backward or forward, in any

other direction than R, possibly in the same direction as R. Such phenomena are necessary results of the relativity of existence, and we must bear them in mind when confronted with complicated conditions which present themselves, for instance in astronomical cases. Here the mistakes rising from the fiction of assuming our reference point to be stable are considerable enough to enforce attention, and in that case we shall have to make allowance for the instability of our reference point, as well as for the time which the rays of light need for their travel through space.

That was exactly Bradley's case as set forth in his essay written in 1727, one hundred and eighty-five years ago, and thus he became the forerunner of the relativity physicists. To state it in other terms, Bradley correctly solved a problem which in our days led to the formulation of the principle of relativity, and he did so without mentioning this theory, yea without feeling the need of formulating it. He simply took it for granted that he had in this case to consider the motion of the earth that served him as a reference point—the place of his observations.

COMSTOCK ON RELATIVITY.

The most popular and at the same time the most exact characterization of the principle of relativity comes from the pen of Prof. D. F. Comstock, of the Massachusetts Institute of Technology. It appeared in *Science* (Vol. XXXI, 1909, p. 767), and we quote from it the passages which contain the statement of the problem:

Professor Comstock starts with the following two postulates:

> "The uniform translatory motion of any system can not be detected by an observer traveling with the system and making observations on it alone.
>
> "The velocity of light is independent of the relative velocity of the source of light and observer."

The main passages of his exposition state the problem thus:

"The whole principle of relativity may be based on an answer to the question: When are two events which happen at some distance from each other to be considered simultaneous? The answer, 'When they happen at the same time,' only shifts the problem. The question is, how can we make two events happen at the same time when there is a considerable distance between them.

"Most people will, I think, agree that one of the very best practical and simple ways would be to send a signal to each point from a point half-way between them. The velocity with which signals travel through space is of course the characteristic 'space velocity,' the velocity of light.

"Two clocks, one at A and the other at B, can therefore be set running in unison by means of a light signal sent to each from a place midway between them.

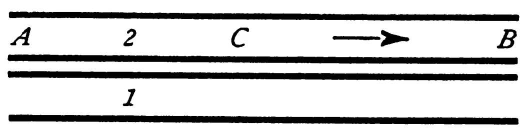

Fig. 2.

"Now suppose both clock A and clock B are on a kind of sidewalk or platform moving uniformly past us with velocity *v*. In Fig. 2 (2) is the moving platform and (1) is the fixed one, on which we consider ourselves placed. Since the observer on platform (2) is moving uniformly he can have no reason to consider himself moving at all, and he will use just the method we have indicated to set his two clocks A and B in unison. He will send a light flash from C, the point midway between A and B, and when this flash reaches the two clocks he will start them with the same reading.

"To us on the fixed platform, however, it will of course be evident that the clock B is really a little behind clock A, for, since the whole system is moving in the direction of the arrow, light will take longer to go from C to B than from C to A. Thus the clock on the moving platform which leads the other will be behind in time.

"Now it is very important to see that the two clocks are in unison for the observer moving with them (in the only sense in which the word 'unison' has any meaning for him), for if we adopt the first

postulate of relativity, there is no way in which he can know that he is moving. In other words, he has just as much fundamental right to consider himself stationary as we have to consider ourselves stationary, and therefore just as much right to apply the midway signal method to set his clocks in unison as we have in the setting of our 'stationary clocks.' 'Stationary,' is, therefore, a relative term and anything which we can say about the moving system dependent on its motion, can with absolutely equal right be said by the moving observer about our system.

"We are, therefore, forced to the conclusion that, unless we discard one of the two relativity postulates, the simultaneity of two distant events means a different thing to two different observers if they are moving with respect to each other."

We quote further:

"It must be emphasized that, because of the first fundamental postulate, there is no universal standard to be applied in settling such a difference of opinion. Neither the standpoint of the 'moving' observer nor our standpoint is wrong. The two merely represent two different sides of reality. Any one could ask: What is the 'true' length of a metal rod? Two observers working at different temperatures come to different conclusions as to the 'true length.' Both are right. It depends on what is meant by 'true.' Again, asking a question which might have been asked centuries ago, is a man walking toward the stern of an eastbound ship really moving west? We must answer 'That depends' and we must have knowledge of the questioner's view-point before we can answer yes or no."

The question of the man walking on a ship not only "might have been asked centuries ago," but it has been asked centuries ago. Our forebears were more conscious of the relativity of existence than the relativity physicists credit them.

Professor Comstock continues:

"It must be remembered that the results of the principle of relativity are as true and no truer than its postulates. If future experience bears out these postulates then the length of the body, even of a geometrical line, in fact the very meaning of 'length,' depends on the point of view, that is, on the relative motion of the observer and the object measured."

Professor Comstock's verdict of the case is summarized in this paragraph:

"The results of the principle for uniform translation are simply as true as its two postulates. If either of these postulates be proved false in the future, then the structure erected can not be true in its present form. The question is, therefore, an experimental one."

Here we demur. We claim that the question is not experimental but belongs to the department of *a priori* reasoning.

Professor Comstock does not enter into questions of mass connected with the principle of relativity but is satisfied with this comment:

"The apparent transverse mass is, I think, best derived by Lewis and Tolman,[3] in their excellent paper on the principle of relativity, and the relation between transverse and longitudinal mass is shown in the most direct and simple way by Bumstead[4] making use of the torsion pendulum. Any one interested in the subject should read these two papers."

THE A PRIORI.

It is characteristic of modern science to denounce the principle of the *a priori* and to extol experiment and experience. Now it is true that experience and experiment are indispensable factors in science, and in all the specialties of science. In experience and experiments we deal with the facts presented to us by nature; but the method of reasoning is not a thing which is derived from sense experience.

The method of reasoning is, as Kant truly said, *a priori* and, let us add, the *a priori* is nothing mystical or mysterious; it is simply the result of pure thought or reflection from which the data of the senses have been excluded. Pure thought (or better, purely formal thought) is a mental construction, or, if you prefer, a fiction. We omit every-

[3] *Phil. Mag.*, 18, 510-523, 1909.

[4] *Am. Jour. of Science*, 26, pp. 493-508, 1909.

thing concrete and thus we retain a field of abstract possibilities. Elsewhere we have called it a field of anyness.[5] Obliterating in our mind all particularity we retain nothing concrete and in this field of nothingness we build up pure relations. From this domain all real things, comprising everything which we subsume under the categories of matter and energy, has been excluded. But these pure relations, i. e., pure forms which are non-material constructions lacking all concrete qualities such as all real things possess, serve us as models for the relations of any possible purely mental or actual existence. Our doings in this field of abstraction consist in the fiction of pure lines, pure numbers, pure motion, pure ideas and their interrelations such as genera and species, and thus we are capable of building up a world of purely formal or relational thought, the totality of which in space is called geometry, and in the domain of numbers which originate by counting a series of single units, arithmetic, etc. In the domain of pure thought, consisting of genera and species, we call the laws that govern their relations logic, and the law of transformation, of which the positive aspect is properly called causality, and its negative counterpart the law of conservation of matter and energy, has been called by Kant pure natural science.

All systems of mental constructions have the advantage of picturing in our mind *any* possible configuration of relativity, and in this sense pure thought (Kant's *a priori*) is a field of anyness. It can be applied to any fact or set of facts of existence, actual or fictitious, and these systems of mental constructions therefore furnish us with the key to determine the relations of real nature. They render possible the systematization of sense impressions and thus

[5] See *Philosophy of Form*, the chapter on "The Foundation of Mathematics and Logic," pp. 7-10. For further details see also the chapter "Form and Formal Thought" in the author's *Fundamental Problems*, pp. 26-60.

these systems of pure thought in the field of anyness are the methods of scientific operation.

Let us not therefore speak contemptuously of the *a priori,* or denounce apriorism as something medieval and elusive, for even here in the attempt at establishing the principle of relativity in time and space, the arguments of the physicists are absolutely aprioristic. There is not one of these so-called experiments, invented to prove the relativity of time and space, which does not ultimately resolve itself into a machine that renders visible aprioristic considerations.

The ultimate arguments in all the experiments made to prove the relativity of time and space move in a domain of purely formal thought, and the force of them is ultimately of the same kind as the Q. E. D. of Euclidean theorems. We think here mainly of such propositions as locate an observer on the sun and another on the earth. Their clocks actually agree, but when compared they are found to differ. About eight minutes have elapsed when the observer on earth registers the time as the rays of the sun reach the earth, and *vice versa* when the clock on earth is observed as the rays from the earth strike the sun. The imitation of the same conditions for the sake of comparing the registration of two moving systems in an actual experiment amounts to nothing more than the pencil drawings of a Euclidean or logical figure in which the *a priori* reasoning is visibly presented as a *demonstratio ad oculos.* The argument remains in either case one of pure thought.

The photograph of such an apparatus built for the purpose of making an experiment in the relativity of time and space to show the difference between a solar clock and a terrestrial clock may be found in the article of Emil Cohn of Strassburg, "Physikalisches über Raum und Zeit" in *Himmel und Erde,* Vol. XXIII. To be sure the instrument does not fulfil the conditions either of distance or of

the velocity of the transference of the signal, "but," says Professor Cohn, "that is of secondary importance."

There are two motions both constant and both standing in a definite proportion. The sun with its clocks has been made to stand still. The earth with its two clocks moves, and there is an arrangement by which to represent the transference of signals. The main thing is that "their velocities stand in definite proportions and all that concerns us are these proportions. That we have here replaced the enormous velocity of light by a velocity of a few centimeters per seecond is unessential. It is essential, however, that the velocity of the earth is three-fourths the velocity of light, while the real ratio is 1 : 10,000."

Newton's laws are *a priori,* and Newton proves that these laws hold good in, and are serviceable as, interpretations of the actual world of fact. The empiricist ought to rebel against Newton's laws, because they never have been nor ever can be proved by either experience or experiment. Whoever saw a body moving in a straight line? and has Newton (from the standpoint of the empiricist) any right at all to make such sweeping statements of movements which have never occurred in the experience of anybody?

The most general principle at the bottom of scientific work is perhaps the so-called law of the conservation of matter and energy, and even this law is based on purely *a priori* arguments.

Incidentally we will say that the law does not hold good if we restrict the notion of matter to matter in the sense of the physicist which is mass, i. e., to concrete particles of existence that are extended and possess weight. It holds good only if we understand by matter the substance of being, its objective reality. We had better therefore speak of the conservation not of matter but of substance, for gross matter, consisting of the chemical elements, is constantly being produced before our eyes in the starry

heavens where the astronomers can watch the process through their telescopes. In the nebulas we see now the commotion of whirls with which gradually first the lighter and then the heavier chemical elements are being manufactured out of the original world-substance which we assume to be the same as the luminiferous ether.

Therefore we may surrender the law of conservation of gross matter, but we still hold to the conception that there is a conservation of stuff or substance, and the same is true of energy. There may be energy in the shape of a stress incorporated in the same wonderful world stuff, the ether, and this stress may be set free and become actual motion or kinetic energy, by some cause which creates those whirls that start the formation of nebulas.

And what proves the law of this conservation of substance and energy? It is the necessity of *a priori* thought which compels us to assume the principle that nothing originates from nothing and nothing disappears into nothing, which thought rests ultimately on the idea that all processes of existence are transformations. Everything that originates is formed by combination from something that existed before.

It has been maintained that the principle of relativity must be proved experimentally, but this is a mistake. Reality is everywhere a system of interrelations, yea every single concrete thing, every phenomenon, every piece of existence is a bundle of relations. It can be analyzed into its elements, which are actions and reactions; and that is all that reality means. Space as well as time are merely the measures, the former of arrangement or position, the latter of succession. Space denotes the interrelation of parts constituting figures or shapes affording a mode of determining direction and distance. Time measures the duration of events which is done by counting uniform cyclical motions or parts thereof. And so we must grant

that the relativity of time and space, as well as of all real things is a universal and inalienable condition of all existence. We can not think of any actuality which would not be dominated by relativity; which means we must regard the principle of relativity as an *a priori* postulate.

The principle of relativity is not established by experience but is ultimately based upon reflection and pure ratiocination. It belongs to the category of purely formal thought as much as all arithmetical and geometrical propositions.

If any proposition of purely formal thought, such as $2 \times 2 = 4$, does not hold good in our experience, we doubt the correctness of our counting or measuring, but we do not doubt our *a priori* proposition. We revise our observation, not our logic, our arithmetic, our mathematics; and suppose our observation proves true, suppose that 2×2 rabbits shut up in a cage are on recounting their number found to be more than four, say six or ten or any higher amount, we do not upset our arithmetic or any of our purely formal propositions, but seek the cause of the irregularity in the objects, in the things or animals counted. In that case we are positive that some transformation of the concrete material has set in which adds to the number to be expected according to arithmetical law.

If the reference point (R) belongs to the same system of motion as the object observed (O), our measurement will be correct and indicate the size of the object adequately. But if R moves in a direction and with a velocity of its own, different from O, the measurement will not be adequate; it will be warped in an exact proportion to the motion of R, and this rule holds good in the same way as all mathematical, logical and generally purely formal theorems.

The reliability of purely formal truths is not merely theoretical, but finds its application in practical life, in the

objective world of matter and motion, and can be verified by experience and experiment. And this is true also of the relativity of time and space.

If for instance a photographer takes the picture of a rapid express train in motion with a camera provided with a curtain shutter, the wheels will not be round but oval in the photograph, and the relativity photographer who identifies the picture with the thing, in the same way as the relativity physicist identifies the result of measuring with the objective size of the object measured, will claim that in proportion to the velocity of the train times the inverse proportion of the velocity of the slit in the curtain of the shutter, the wheels will increase their horizontal diameters and become that much more oval. Yea they will insist that the very same wheel will be at the same time in one camera, only a little more, in another one much more oval according to the quickness with which the slit of the curtain passes over the sensitive plate.

The relativity photographer will claim that the wheels in motion *are* oval while common mortals think that they only appear oval in the photograph.

Photographs do not lie; they show the objects photographed without any personal equation on the part of the photographer; their objectivity and impartiality can not be doubted, and here we see the wheels oval. They are oval, and their ovality, viz., their deviation from true circles, depends on the velocity of certain motions. An enthusiast for the principle of relativity can justly claim that every photograph of a rapid train which shows the oval form of the wheels is a successful experiment in the demonstration of the relativity of figure in space.

The truth of the principle of relativity in the domain of photography can be explained by *a priori* considerations. It is a matter of course, and if we argue the subject in our mind in pure reflection, we find out what we must expect,

and if finally we make the experiment, the principle proves true.

In the same way all the experiments made by machinery so constructed as to represent terrestrial and solar clocks or yard sticks, and to point out the unavoidable difference of measurements in both time and size resultant from their respective motions of the earth and the sun as well as the time it takes to transmit signals, are not experiments in the physicist's sense but expositions and demonstrations of purely formal truths which belong to the category of mathematics.

If the principle of relativity does not hold good in any domain of actual life, we must seek the cause in the material used and not in the principle of relativity. In other words we would be confronted with a purely physical problem which demands a physical solution, and this seems to be the case of the Fizeau experiment.

Prof. Emil Cohn, of Strassburg,[6] says:

> "It is strange that the relativity principle of mechanics does not hold good in radiation—in radiation and therewith in electrodynamics, for that the spread of radiation is an electrical process we may consider since Heinrich Hertz as an assured matter of experience. The decisive experiment which has been made by Fizeau is this: In a liquid, flowing with a uniform velocity, light is to be propagated in the direction of the current. According to the relativity principle an observer drifting in the current should find the velocity of propagation to be the same as if the liquid were at rest, and an outside observer should find the velocity of the light augmented by the full velocity of the current in the liquid. (Think, e. g., of the ball rolling on the deck of a ship in motion.) But such is not the case. There is added only a certain portion, viz., the index of refraction."

The very result of the experiment proves that one of the determinant factors is the physical property of the fluid.

When the principle of relativity is applied to positive

[6] *Loc. cit.*, p. 7.

facts we reach slippery ground, on which we must be on our guard to avoid mystification, for it would seem as if the law of the conservation of matter and energy were upset and all objectivity of scientific truth were lost. Experiments have been made to prove the principle of relativity with the result that Hupka and Bucherer,[7] the former with cathode rays, the latter with radium rays, demonstrate that mass increases with velocity as the relativity principle demands. Kaufmann, however, comes to the conclusion that there is an increase of mass but not as ought to be expected according to the principle of relativity, while Michelson and Morley demonstrate with great exactness that in spite of the motion of the earth the transmission of light is not changed at all, not within one hundred millionth of its proportion nor even a fraction thereof.

It would lead us too far to discuss the experiments made to apply the principle of relativity to physics and electrodynamics; we will only mention that (as *a priori* might be expected) they tend to corroborate its applicability in these domains.

ON ABSOLUTE MOTION.

Dr. Philipp Frank in his discussion "Does Absolute Motion Exist?"[8] declares that motion in physics always means "motion with reference to some definite body," and he recognizes that "this question is a philosophical one[9] but it is certainly not a physical question." The answer is the first Newtonian law, viz., "A body not affected by an exterior force moves in a straight line with a constant

[7] A. H. Bucherer, "Die experimentelle Bestätigung des Relativitätsprinzips" in *Annalen der Physik*, XXVIII, p. 513; "Messungen an Becquerelstrahlen" in *Physikalische Zeitschrift*, IX, pp. 755-760.

[8] "Gibt es eine absolute Bewegung?" Lecture delivered December 4, 1909, at the University of Vienna before the Philosophical Society. *Wissenschaftliche Beilage*, 1910.

[9] Dr. Frank adds here: "Perhaps the psychologist would call it a psychological one," but this would be a mistake. Psychology has nothing to do with the subject.

velocity which of course may be zero.[10] This is called the law of inertia."

If another force affects the moving body it is subject to the second law, the law of the parallelogram of forces, according to which the body will move along the diagonal of the two forces.

The following extracts translated from Dr. Frank's essay on absolute motion will prove instructive:

"The system of the fixed stars constitutes a fundamental body. Even in shooting a cannon ball towards the south we see no deviation from the law of inertia if we consider it with reference to the fixed stars. The ball remains in the same plane; but this plane does not retain the same relative position to the meridian of the earth, wherefore, of course, with reference to the earth the law of inertia is violated. On the whole it is evident that we really recover all the observed motor phenomena when we refer Newton's laws of motion to the fixed stars. Not until they are referred to the fixed stars do these laws acquire an exact sense which makes it possible to apply them to concrete conditions.

"We shall call those motions which are referred to a fundamental body 'true movements' and those related to any other body of reference 'apparent movements.' For instance the immobility of my chair is only apparent, for when referred to the fixed stars it is in motion.

"We now ask whether there are any other fundamental bodies aside from the system of the fixed stars. Obviously not any body revolving in an opposite direction to the fixed stars can be such a fundamental body, for considered with reference to such a body all rectilinear movements are curved. Therefore the law of inertia could not hold with reference to the body in question if it is valid with reference to the fixed stars. Then too a fundamental body can possess no acceleration with reference to the fixed stars, because otherwise there would be no uniformity of the motion of inertia with reference to it. However, these conditions are not only necessary but they are sufficient to characterize a fundamental body. All bodies moving uniformly and in a straight line with reference to the fixed stars will also be fundamental bodies inasmuch as rectilinearity and

[10] The original reads thus: "Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum nisi quatenus a viribus impressis cogitur statum illum mutare."

uniformity continue to hold for them, as do likewise the supplementary velocities determined by the second law. Accordingly Newton's laws do not indicate one single fundamental body, but an infinite number moving in opposite directions with a uniform and rectilinear motion.

"Hence we may well speak of 'true' in contrast to apparent rotary motion; for all bodies revolving with reference to a fundamental body revolve with reference to all other bodies. The same is true of true acceleration because an acceleration with respect to a fundamental body is also acceleration (i. e., change of velocity) with respect to all the rest. On the other hand, there is no sense in speaking of 'true' uniform rectilinear motion; for if a body possesses a uniform velocity with respect to the fixed stars, it is itself a fundamental body possessing of course with respect to itself a velocity of zero; it is at rest.

"Accordingly there is true acceleration, but not true velocity. From this is easily derived a proposition established by Newton which is called the principle of relativity of mechanics, namely that a uniform rectilinear movement of the system as a whole makes no change in the processes within the system; that is to say, we can not tell from the processes within the system what velocity the uniform rectilinear movement possesses with reference to the fixed stars. On the other hand, the rotary motion of a system has indeed an influence on the processes within the system, as for instance in the phenomena of centrifugal force; thus the earth has become flattened at its poles because of its rotation, or if I revolve a dish full of water the water will rise at the sides."

ABSOLUTE SPACE.

If we make measurements of motions which are limited to terrestrial conditions, the earth is and must be the system which, though not absolute, must for the nonce be so considered, and in that case the earth is called the fundamental or inertial body, of our measurements. But in many purely terrestrial motions we observe in very precise and exact measurements, deviations which compel us to seek for another fundamental body.

This happens in the case of the Foucault pendulum experiments and may also be observed in a cannon ball which

if shot south along the meridian will at a great distance show a deviation toward the west. Such experiments point out that the entire system of the fixed stars ought to be regarded as the fundamental body which thus would represent to us absolute space. I say here on purpose "represent to us," not "be," because we are most probably in the same predicament as persons moving in a train to whom the train and its interrelations, so long as the train does not move in a curve, represent the fundamental body or absolute space, viz., the ultimate system of reference.

It stands to reason that bodies in translation (in which the entire system as a whole moves in the same direction with the same velocity and without any internal change even of its smallest particles) behave as if they were at rest, and so the motion of a straight line cannot be observed so long as the observer remains limited to his own system. Every deviation from a straight line, however, implies a retardation on the inner side of the curve, or, what means the same, an acceleration on the outside of the curved path of motion. Accordingly all rotations bear witness to the character of their motion as appears in the Foucault pendulum experiment and in the flattening of the earth at the poles. Since further the idea of a rectilinear motion is a mere *a priori* postulate which can never be realized in actual nature, we see that every motion that takes place anywhere is affected by the totality of the universe. We must assume that its existence (the existence indeed of every particular thing or the recurrence of any event) must be understood to be a part of the whole. It bears traces of all the influences of all masses, and of all forces of the rest of the world according to the way it is interrelated with its surrounding conditions.

The fixed stars have so far proved sufficient for our terrestrial needs to serve us as a fundamental body for

calculations of a mechanical nature; but here the problem of absolute space presents itself.

We know positively that though the fixed stars are practically a fundamental body to us for mechanical measurements, they are shifting about among themselves and no more constitute something absolute than does our own earth; and yet there has risen a controversy on this subject in which Ernst Mach applies the principle of relativity throughout the universe while Prof. Alois Höfler stands up for what he calls the absolutist theory. We will hear what Dr. Frank has to say on this point:

"Is it to a certain extent accidental, or is it essential, that the tatality of the fixed stars coincides with that fundamental body in relation to which the laws of Newton hold valid? Or to put it more clearly: If the fixed stars were set violently in motion among each other and hence could no longer constitute a fixed body of reference, would the mechanical processes on earth proceed exactly as they did before? For instance, would the Foucault pendulum move just as at present, even though it now turns with the fixed stars, whereas in that case it would not be quite clear which constellation's revolution it should join?

"Were everything to remain as of old the fundamental system of reference would not be determined by the fixed stars but would only accidentally coincide with them, and would in reality be some merely ideal or yet undiscovered body. In the other case all mechanical occurrences on earth would have to be completely altered to correspond with the promiscuous movements of the fixed stars.

"It is well known that this is the view held by Ernst Mach. It alone holds with consistent firmness to physical relativism, and it alone answers the second main question of physics in the relativistic sense.

"The opposite view is represented by Alois Höfler in his studies on the current philosophy of mechanics, and lately by G. Hamel, professor of mechanics at the technical high school of Brünn, in an essay which appeared in the annual report of the German mathematical society of 1909 on 'Space, Time and Energy as *a priori* Forms of Thought.'

"Before I enter upon the controversy itself I would like further

to elucidate Mach's view by carrying out its results somewhat farther. In his well-known essay on the *History and Root of the Principle of the Conservation of Energy*[11] Mach ascribes to the distant masses in space a direct influence on the motor phenomena of the earth which supplements the influence afforded by gravitation. Of course no effect of gravitation from the fixed stars upon the earth can be observed, yet in spite of this they influence, for instance, the plane of oscillation of the Foucault pendulum because in Mach's opinion it remains parallel to them.

"The question now arises according to what general law of nature this influence operates which does not, like gravity, produce accelerations but velocities instead. Obviously this influence must be a property belonging to every mass, for according to our present conception the fixed stars of course are precisely the same sort of masses as earthly bodies.

"However, experience teaches us that terrestrial masses have no more influence on the plane of oscillation of the Foucault pendulum than has the changing position of the moon, sun and planets; but on the other hand it is exactly the most distant masses, the fixed stars, which determine its plane of oscillation. Accordingly we must either assume that the effect is directly proportional to the distance of the masses (which would be very strange indeed) or simply assume that this effect is proportional to the effective masses and independent of the distance, whence the dominant influence of the more remote, as the far greater and more numerous, bodies would naturally follow, and Mach inclines to this latter view.

"Mach's view shows most clearly in his position with regard to Newton's famous bucket experiment. In this Newton intended to show that the centrifugal force produced by a revolving body is due not to its relative but to its absolute velocity of rotation. He suspended a bucket filled with water by a vertical cord, twisted the cord quite tightly and then let it untwist itself, in this way setting the bucket to revolve rapidly. At first the water did not rotate with the bucket and therefore the bucket had a velocity of rotation with reference to the water while in the meantime the surface of the water remained undisturbed. In time, however, friction caused the water to become so affected by the rotary motion that bucket and water revolved like one homogeneous mass whereby the centrifugal

[11] Second edition, Leipsic, 1909; English translation by P. E. B. Jourdain, Chicago, 1911.

force caused the water to rise at the sides of the bucket and the surface became concave.

"Hence it is evident that the centrifugal force reached its greatest strength at the moment when the relative motion of the water with respect to the bucket became zero; hence according to Newton this force can be produced only by the absolute rotary motion of the water.

"To this now Mach justly protests that only the relative rotation of the water with reference to the fixed stars is to be considered, for this system of the fixed stars and not the bucket is the fundamental body. And indeed at first the water was at rest with reference to the fixed stars, but at the close of the experiment it was revolving. The mass of the bucket compared to the mass of the fixed stars is an entirely negligible quantity, so that it does not depend in the least upon the rotation. But we can not know, adds Mach, how the experiment would turn out if the sides of the bucket were miles thick; and by this he apparently means so thick that their mass would be considerable even when compared with the mass of the system of fixed stars. Then indeed might the rotation of the bucket disturb the action of the fixed stars.

"Höfler protests, on the other hand, that a system which is symmetrical round its axis could not according to all our experience in mechanics produce by its rotation that sort of an effect on the water within it.

"This also is quite true. But the effect of the masses assumed by Mach is such that it can not be expressed in our ordinary experiences with mechanics except by means of the facts of the inertia of all motion with reference to the fixed stars. New conditions such as the rotation of an enormously thick bucket might give rise to new phenomena. If we agree with Mach's view that the rotation of the plane of the Foucault pendulum is directly produced by the masses of the fixed stars, we must likewise admit, in order to be consistent, that the relative rotation of the very thick bucket might give rise to similar effects with reference to the water, as the rotation of the system of the fixed stars with reference to the earth to the plane of oscillation.

"Höfler expresses his contention against Mach's thesis in the form of the following question: If in Galileo's time the sky had been clouded over and had never become clear again so that we would never have been able to have taken the stars into our calculation, would it then have been impossible to have established our

present mechanics solely by the aid of terrestrial experiments? By this question Höfler means to say that if the connection with the fixed stars were a constituent of the concept of uniform motion, we would never have been able in such an overclouded world to have established the law of inertia, for instance, whereas in reality it is clear that this would nevertheless have been possible.

"I will not dwell on the more psychological question as to whether or how easily this would have been possible, but will only consider now the logical construction of mechanics in such a darkened world on the hypothesis that easily or with difficulty in one way or another we would have attained to our present knowledge of mechanics.

"Let us for a moment imagine ourselves in such a world. Above our heads extends a uniform vault of uninterrupted gray or black. Were we to shoot projectiles toward the south we would see that they describe paths which are curved towards the west; if we started pendulums to vibrating we would see that they would revolve their planes of oscillation in mysterious periods—I say mysterious because we might perhaps be able to perceive the change of day and night as an alternation of light and darkness, but would not be able to refer it to the movements of celestial bodies. Perhaps at first we would surmise that the motion of the pendulum could be ascribed to optical influences. I would like to see placed in such a world one of the philosophers who regard the law of inertia as an *a priori* truth. In the face of these mysterious curvatures and deflections he would probably find no adherents and he would not know himself what to make of his own standpoint.

"Finally, let us assume, there arises a dauntless man, the Copernicus of this starless world, who says that all motions proceed spontaneously in a straight line, but that this straight line is not straight with reference to the earth but with respect to a purely ideal system of reference which turns in a direction opposite to that of the earth. The period of this rotation is supplied by the period of the Foucault pendulum.

"This man would of course deny physical relativism upon the earth, for in his opinion terrestrial processes would not depend only on the relative velocities of terrestrial bodies but on something else besides, viz., their velocities with respect to a purely ideal system of reference. Nevertheless, he would not introduce any nonphysical element because for the purpose of the physicist a purely ideal system of reference whose motion with respect to an em-

pirical system is known serves the same purpose as would the empirical system itself. This bold innovator might finally refer the words 'true rest' and 'true motion' to his ideal fundamental body and so ascribe true motion and only apparent rest to the earth, thus maintaining a mechanics which would coincide literally with that of ours to-day, except that no small luminous points would be seen sparkling in connection with the fundamental body.

"Hence we see that physical relativism is not a necessary tool of the physicist. Apart, perhaps, from the psychological improbability—of which, however, nothing more positive can be said—the possibility of the development here indicated is logically free from objections throughout, and therefore the same is also true of the possibility of a nonrelativistic physics.

"But I would like to strengthen the argument of Höfler even somewhat further. That is to say, I would ask whether the world in which we live is then really so essentially different from that fictitious one. Imagine the dark roof which conceals the sky placed somewhat higher so that there is room beneath it for the fixed stars, perhaps as the dark background which may be seen nightly in the starry sky. The whole difference then consists in the fact that not only the Foucault pendulum and similar appliances move with reference to the earth, but enormously greater masses as well—all the twinkling lights of the sky by which the thought of a fundamental body in motion with respect to the earth is psychologically greatly facilitated, but logically is not much changed. Now imagine the sky of this earlier dark world suddenly illuminated; then we would see that the fictitious system of reference is closely linked to enormous cosmic masses, and it would be easy enough to accept Mach's hypothesis that these masses condition the fundamental system....

"If a distinction must be drawn between the respective values of the conceptions of Mach and Höfler, it is as follows: Mach's view adds decidedly more to the observed facts; for that it retains physical relativism does not involve freedom from hypothesis, because at best this relativism is theory and not fact. Mach sets up, hypothetically of course, a new formal natural law with regard to the action of masses existing side by side with gravitation, affecting the experiment very materially but unable to raise any claim to the simplest description of actual conditions.

"The other view, which simply introduces the system of reference procured by observation of the terrestrial and celestial movements without asking whence all this is derived, represents the pres-

ent state of our knowledge most adequately without any arbitrary addendum but also without giving the spirit of inquiry any incentive to new experiments.

"It is the old contrast between the most exact and least hypothetical representation possible of the known science, and progressive inquiry after new things in more or less daring and fantastic hypotheses. But Mach in this case stands in the opposite camp as in most other cases where his repugnance to all hypothesis has made him a pioneer in the phenomenological direction....

"I therefore believe I have proved that we can grant the following: Physical phenomena do not depend only on the relative motion of bodies without at the same time admitting the possibility of the concept of an absolute motion in the philosophical sense."*

Strange that Mach, with his reluctance to introduce anything hypothetical except what is absolutely indispensable, should range on the side of the theorists, and after some reflection I believe that there may be a slight hitch in Dr. Frank's interpretation of Mach's view.

First I myself, from my own point of view, would refuse to call the principle of relativity an hypothesis; it is an *a priori* proposition, a theorem, or if you prefer, a postulate of pure thought which either holds good universally, or has no validity whatever. So far as I know, Mach has not discussed this side of the subject but he has instinctively acted upon this view, and I would say that there is a greater hypothetical element in the assumption that the theorem $2 \times 2 = 4$, or any other proposition of the same kind, holds good only for our earth but not for Mars and Venus, than to say that it holds good also for the fixed stars and in the possible worlds outside of our Milky Way. Accordingly, whatever Mach's personal opinion may be, I would regard the universal application of the principle of relativity as less complicated and more free from hypo-

* This last paragraph is printed in spaced letters which indicates the emphasis of the author, and so we print the text of his summary in the original. Dr. Frank says: "Die physikalischen Erscheinungen hängen nicht nur von der Relativbewegung der Körper ab, ohne doch damit die Möglichkeit des Begriffes einer absoluten Bewegung im philosophischen Sinne zuzugeben."

thetical elements than its limitation to a portion of the world.

I can not as yet make up my mind to believe that our system of the Milky Way which furnishes us the grand sight of the fixed stars is an ultimate possessing the characteristics of absolute space.

According to Kant the totality of the fixed stars which are thickest in the Milky Way forms a great system (the system of the Milky Way) and our sun as well as all the visible fixed stars belongs to it. Kant believes that this, our own universe, which in the Milky Way appears to us as an enormous ring but together with the totality of the fixed stars must resemble an oblate spheroid, is not the only cosmic system, but that there are other similar systems outside of it and that they too whirl on through the infinity of space, in company with our Milky Way system, around some center of their own; and this very center of many Milky Ways may partake of a motion the observation of which lies hopelessly beyond our ken. Accordingly the space conditions of the Milky Way may serve *us* as absolute space, but there is a probability that this space is not more absolute than are the space relations in a quick but quietly moving train to the passengers.

Another point where we feel justified in doubting Dr. Frank's exposition is the statement that Mach hypothetically assumes a new law of nature as to the efficacy of masses, besides the law of gravitation. The passage in Mach's writings to which Dr. Frank refers does not (in my opinion) suggest the idea of an additional law of nature according to which the distant fixed stars should exercise a mysterious influence on the Foucault pendulum. We will later on let Mach speak for himself. In our opinion it seems that it would be sufficient to ascribe the rotation of the pendulum to its inertia while the earth revolves round itself, and this takes place in the space in which the earth

has its motion, viz., the space of the Milky Way system. The pendulum remains in the plane of oscillation in which it started while the earth turns around underneath. If there are influences at work beyond the expanse of the space of the fixed stars in our Milky Way system, they must affect the totality of our system and would therefore be contained in its space conditions; acting with an unfailing constancy they could not be separated from the properties of our space and would scarcely be discoverable.

There seems to me no need of inventing a new force besides gravitation. The law of inertia seems to explain the Foucault pendulum experiment satisfactorily.

The fixed stars as a totality remain in their places (at least as far as concerns the experiment) and the plane in which the pendulum swings keeps its original direction; thus the apparent motions of both coincide. Their space relations (the space relations of the pendulum and of the fixed stars) are the same, and there is no need to assume the existence of any unknown force exercised by the fixed stars upon the pendulum.

ERNST MACH.

We will let Mach state his views in his own words:

"Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. Geometrically these are exactly the same case of a relative rotation of the earth and of the celestial bodies with respect to one another. Only, the first representation is astronomically more convenient and simpler.

"But if we think of the earth at rest and the other celestial bodies revolving round it, there is no flattening of the earth, no Foucault's experiment, and so on—at least according to our usual conception of the law of inertia.

"Now, one can solve the difficulty in two ways: Either all motion is absolute, or our law of inertia is wrongly expressed. Neumann[12] preferred the first supposition, I, the second. The law of

[12] *Ueber die Principien der Galilei-Newton'schen Theorie.* Leipsic, 1870.

inertia must be so conceived that exactly the same thing results from the second supposition as from the first. By this it will be evident that, in its expression, regard must be paid to the masses of the universe.

"In ordinary terrestrial cases, it will answer our purposes quite well to reckon the direction and velocity with respect to the top of a tower or a corner of a room; in ordinary astronomical cases, one or other of the stars will suffice. But because we can also choose other corners of rooms, another pinnacle, or other stars, the view may easily arise that we do not need such a point at all from which to reckon. But this is a mistake; such a system of coordinates has a value only if it can be determined by means of bodies....

"If we wish to apply the law of inertia in an earthquake, the terrestrial points of reference would leave us in the lurch, and, convinced of their uselessness, we would grope after celestial ones. But, with these better ones, the same thing would happen as soon as the stars showed movements which were very noticeable. When the variations of the positions of the fixed stars with respect to one another cannot be disregarded, the laying down of a system of coordinates has reached an end. It ceases to be immaterial whether we take this or that star as point of reference; and we can no longer reduce these systems to one another. We ask for the first time which star we are to choose, and in this case easily see that the stars cannot be treated indifferently, but that because we can give preference to none, the influence of all must be taken into consideration.

"We can, in the application of the law of inertia, disregard any particular body, provided that we have enough other bodies which are fixed with respect to one another. If a tower falls, this does not matter to us; we have others. If Sirius alone, like a shooting star, shot through the heavens, it would not disturb us very much; other stars would be there. But what would become of the law of inertia if the whole of the heavens began to move and the stars swarmed in confusion? How would we apply it then? How would it have to be expressed then? We need not worry about one body as long as we have others enough. Only in the case of a shattering of the universe we learn that all bodies, each with its share, are of importance in the law of inertia....

"Yet another example: A free body, when acted upon by an instantaneous couple, moves so that its central ellipsoid with fixed center rolls without slipping on a tangent-plane parallel to the plane of the couple. This is a motion in consequence of inertia. Here the body

makes very strange motions with respect to the celestial bodies. Now, do we think that these bodies, without which one cannot describe the motion imagined, are without influence on this motion? Does not that to which one must appeal explicitly or implicitly when one wishes to describe a phenomenon belong to the most essential conditions, to the causal nexus of the phenomenon? The distant heavenly bodies have, in our example, no influence on the acceleration, but they have on the velocity."

Now follows the passage to which Dr. Frank obviously refers:

"Now, what share has every mass in the determination of direction and velocity in the law of inertia? No definite answer can be given to this by our experiences. We only know that the share of the nearest masses vanishes in comparison with that of the farthest. We would, then, be able completely to make out the facts known to us if, for example, we were to make the simple supposition that all bodies act in the way of determination proportionately to their masses and independently of the distance, or proportionately to the distance, and so on. Another expression would be: In so far as bodies are so distant from one another that they contribute no noticeable acceleration to one another, all distances vary proportionately to one another."

We do not here understand Mach to fall back on the assumption of a new kind of force, and if we must grant that the distant masses exercise a dominant influence while the influence of the nearest ones (of the earth, the moon, and the sun) vanishes, we would say that this is due to the constancy of the distant masses which, as it were, is an inherent and inalienable part of all mass in the entire system and may be said to characterize its space conditions.

In speaking of "space conditions" I am conscious of using a term which Mach would repudiate, for he claims that for a comprehension of the concatenation of events, the notions of time and space are redundant. He says (*loc. cit.* pp. 60-61):

"To say the least, it is superfluous in our consideration of causality to drag in time and space. Since we only recognize what we

call time and space by certain phenomena, spatial and temporal determinations are only determinations by means of other phenomena. If, for example, we express the positions of earthly bodies as functions of the time, that is to say, as functions of the earth's angle of rotation, we have simply determined the dependence of the positions of the earthly bodies on one another.

"The earth's angle of rotation is very ready to our hand, and thus we easily substitute it for other phenomena which are connected with it but less accessible to us; it is a kind of money which we spend to avoid the inconvenient trading with phenomena, so that the proverb "Time is money" has also here a meaning. We can eliminate time from every law of nature by putting in its place a phenomenon dependent on the earth's angle of rotation.

"The same holds of space. We know positions in space by the affection of our retina, or our optical or other measuring apparatus. And our x, y, z in the equations of physics are, indeed, nothing else than convenient names for these affections. Spatial determinations are, therefore, again determinations of phenomena by means of other phenomena.

"The present tendency of physics is to represent every phenomenon as a function of other phenomena and of certain spatial and temporal positions. If, now, we imagine the spatial and temporal positions replaced in the above manner, in the equations in question, we obtain simply every phenomenon as function of other phenomena.

"Thus the law of causality is sufficiently characterized by saying that it is the presupposition of the mutual dependence of phenomena. Certain idle questions, for example, whether the cause precedes or is simultaneous with the effect, then vanish by themselves."

We understand that Mach endeavors to eliminate the terms time and space, because he wishes to correct the common notion which regards space as a big box into which the world has been packed. Mach says:

"Space and time are not here conceived as independent entities, but as forms of the dependence of the phenomena on one another. I subscribe, then, to the principle of relativity, which is also firmly upheld in my *Mechanics* and *Wärmelehre*."[18]

We agree with Mach. There is no time in itself; there

[18] Cf. "Zeit und Raum physikalisch betrachtet," in *Erkenntnis und Irrtum*, Leipsic, 1905 (2d ed. 1906, pp. 434-448); See also *Space and Geometry*, pp. 94 ff.

is no space in itself. Nevertheless, Mach has given much attention to physical space and appreciates the important part which it plays not only in the formation of our space-conception, but also in the actual world, for every spot of space possesses physical qualities according to the particles of mass which are there aggregated. Mach says:

"Since the positions in space of the material parts can be recognized only by their states, we can also say that all the states of the material parts depend upon one another.

"The physical space which I have in mind—and which, at the same time, contains time in itself—is thus nothing other than dependence of phenomena on one another. A complete physics, which would know this fundamental dependence, would have no more need of special considerations of space and time, for these latter considerations would already be included in the former knowledge."

The same idea is expressed by Mach in his Essay "Ueber den Zeitsinn des Ohres:[14]

"Physics sets out to represent every phenomenon as a function of time. The motion of a pendulum serves as the measure of time. Thus, physics really expresses every phenomenon as a function of the length of the pendulum. We may remark that this also happens when forces, say, are represented as functions of the distance; for the conception of force (acceleration) already contains that of time. If one were to succeed in expressing every phenomenon—physical and psychical—as a function of the phenomenon of pendulum-motion, this would only prove that all phenomena are so connected that any one of them can be represented as a function of any other. Physically, then, time is the representability of any phenomenon as a function of any other one."

We do not deny the truth of Mach's view. Nevertheless time and space are very convenient terms denoting two categories of certain interrelations (he would call them interdependencies) in the flux of things. Popular terms mostly originate because there is a need of them, and it seems to me it would be wiser to correct the errors connected with them than to drop them. If we pursue the

[14] *Sitzb. der Wien. Akad.*, 1865. Compare *Conservation of Energy*, p. 90.

latter policy we shall find ourselves obliged to reinvent a new collective term for certain classes of relations which belong together and can not be identified with other relations. The space and time relations are radically different from those of a purely physical, chemical or psychological nature.

We need not fear to retain the old terms, space and time, if we only bear in mind that there is neither absolute space nor absolute time but that the words denote relations. It seems to me that when Kant speaks of the ideality of space and time and insists on their non-existence as objective beings (*Wesen* or *Wesenheiten*) he attempts to say the same as Mach who declares that they are not "independent entities."

The conclusion at which we arrive in considering the nature of time and of space, be it from our standpoint of philosophy or from Mach's physical point of view, may be expressed in one word, that their most obvious characteristic is relativity.

CONCLUSION.

Professor Mach says in one of his notes quoted above, "I subscribe then to the principle of relativity," and so do I. Indeed I go one step further. I consider relativity as an inherent quality of existence and so I adopt the principle of it not as a result of experience but on *a priori* grounds. The principle of relativity, however, is frequently stated by relativity physicists as if the old ideal of science in its objective significance had to be abandoned, as if physics had to be remodeled, and as if the proclamation of the principle of relativity indicated a new departure from our traditional methods. This is not so, and I must insist that the principle of relativity has always been subconsciously in the minds of scientists. Only it has lately

been forced upon the attention of physicists by the progress in astronomical measurements.

How helpful the emphasis recently laid upon the principle of relativity will prove remains to be seen. Its ardent adherents exhibit great zeal which in many directions seems to be misdirected, and it appears to me that in spite of the correctness of the underlying idea their hopes are greatly exaggerated. After a while when the opponents of the principle of relativity will understand that its truth is as much a matter of course as the truth of the law of conservation of matter and energy, the contentions about it will cease and the evolution of science will no longer show evidence of excitement but will continue in its old quiet way.

There is more philosophy in our science than the school of empiricists are inclined to believe. It is very desirable that in familiarizing themselves with philosophy, these scientists should not fall back on the old systems of a visionary absolute, but they should adopt the philosophy of science, the only philosophy which is not a mere ingenious dream, and possesses objective significance.

The philosophy of science is *the* philosophy. It is the indispensable introduction to the study of any science and furnishes the basis for scientific method as well as a general survey of the assured results of all the several sciences. If the philosophy of science had been better known, the principle of relativity had at once been rightly understood and the vagaries of many mystifying contentions would have been avoided.

* * *

The purpose of this article is to set forth in general outlines the truth and significance of the principle of relativity, not to present an exhaustive treatment of it in all its phases and applications. We must bear in mind that in dealing with the several innumerable problems of exist-

ence science introduces a method which possesses certain limitations due to conditions which originate through some fictions of an apparently arbitrary nature assumed for the sake of isolating the object of investigation and concentrating upon it our attention.

We must bear in mind that we behold an object by focusing our eyes upon it and that only thereby can we form a picture of the object. It is a fiction to behold an object as if it were a thing by itself and it is positively impossible to see anything as it is in all its relations and with all its changes, past, present and future. Nor would such a comprehension of the object in all its entirety be desirable, for in the omneity of its relations we would see the whole universe while the special feature which concerns us sinks into insignificance. The same is true of science. Each of the several sciences selects its own field of investigation and thus constitutes a definite domain of abstraction for the sake of concentrating all attention upon it. For mechanics and for the measurements of motion in space, we need a reference point which must be able to be considered stationary, and if that is not the case we must refer both the movable place of observation, viz., the reference point (R) and the object observed (O) to one common system, which could be treated as, or must so far as R and O are concerned, actually be, stable.

We conclude by repeating that there is nothing absolute; all real and actual existences, all concrete things and happenings are relative, and if there is any thing that in a certain sense deserves the name absolute it is the truth as described in our mental fictions, the laws of purely formal thought, the eternal uniformities of purely formal relations such as we know from mathematics and all the other purely formal sciences; but even they are absolute only in the sense of constituting an entire system the truth of which is absolute, viz., it stands aloof and is founded in it-

self as a world of necessary conclusions built up in the field of anyness to serve as models for any conditions in any world actual or imaginary. And this absolute, this system of mental construction is after all a system of relations.

The more we ponder on the nature of existence, the more we shall understand the sweeping significance of relativity.

P. C.